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Society of Mechanical Ventilation

Equations and Calculations

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Introduction

The human body is a complex, interconnected system where physiological processes are meticulously regulated to maintain homeostasis. Among the most critical of these processes are the respiratory, cardiac functions, and the acid-base balance, each of which plays a vital role in sustaining life. Understanding the underlying principles of these systems requires not only a grasp of their anatomy and physiology but also a familiarity with the mathematical equations that quantify their functions. These equations serve as foundational tools for clinicians, researchers, and students, providing a framework to analyze, diagnose, and treat a wide range of physiological and pathological conditions.

This chapter delves into the key equations governing these systems, exploring their derivation, application, and significance in both physiological and pathological contexts. By mastering these concepts, we gain a deeper appreciation of the intricate mechanisms that sustain life and the tools to intervene when these mechanisms falter.

Respiratory & Cardiovascular Equations

Alveolar Oxygen Gas Equation (PAO_2)

Used to estimate the partial pressure of oxygen in the alveoli (Normal: 104 mmHg at sea level)

$PAO_2 = FIO_2 (Patm - PH_2O) - PACO_2 / R$

- FIO_2: Fraction of inspired oxygen (e.g. 0.21 for room air).
- Patm: Atmospheric pressure (760 mmHg at sea level).
- PH_2O: Partial pressure of water vapor (47 mmHg at body temperature).
- $PACO_2$: Alveolar partial pressure of carbon dioxide (assumed equal to arterial PaCO2).
- R: Respiratory exchange ratio (typically 0.8).

Alveolar Carbon Dioxide Equation ($PACO_2$)

Describes the relationship between alveolar ventilation and alveolar partial pressure of CO_2 (Normal 40 mmHg)

$PACO_2 = VCO_2 / VA \times 0.863$

- VA: Alveolar ventilation
- VCO_2 : CO_2 production
- 0.863: Constant K

Alveolar-Arterial Oxygen Gradient (A-a)

Evaluates gas exchange efficiency (Normal: 10 - 20)

A-a Gradient = $PAO_2 - PaO_2$

- PAO_2: Alveolar oxygen partial pressure (from Alveolar Gas Equation).
- PaO_2: Arterial oxygen partial pressure.

Partial Pressure of Inspired Oxygen (PIO_2)

The oxygen pressure in inspired air (Normal: 90 – 100 mmHg)

$PIO_2 = FIO_2 \times (Patm - PH_2O)$

- PIO_2: Inspired oxygen partial pressure (mmHg).
- FIO_2: Fraction of inspired oxygen.
- Patm: Atmospheric pressure (760 mmHg at sea level).
- PH_2O: Water vapor pressure (47 mmHg at 37°C).

Arterial Oxygen Content (CaO_2)

Determines the total oxygen content (CaO_2) in arterial blood (Normal: 16 – 20 mLO_2/dL)

$CaO_2 = (1.34 \times Hb \times SpO_2) + (0.0034 \times PaO_2)$

- Hb: Hemoglobin concentration (g/dL).
- SpO_2: Oxygen saturation of hemoglobin (as a fraction, e.g. 0.97 for 97%).
- PaO_2: Partial pressure of oxygen in arterial blood (mmHg).
- 1.34: Oxygen carrying capacity of hemoglobin (mLO_2 per g of Hb).
- 0.003: Solubility of oxygen in plasma (mLO_2 per mmHg per dL).

Venous Oxygen Content (CvO_2)

This calculates the oxygen content in venous blood, analogous to arterial oxygen content but using mixed venous parameters (Normal: 12 – 16 mLO_2/dL)

$CVO_2 = (1.34 \times Hb \times SvO_2) + (0.0034 \times PVO_2)$

- Hb: Hemoglobin concentration (g/dL).
- SvO_2: Oxygen saturation of hemoglobin (as a fraction, e.g., 0.75 for 75%).
- PvO_2: Partial pressure of oxygen in mixed venous blood (mmHg).
- 1.34: Oxygen carrying capacity of hemoglobin (mL O_2 per g of Hb).
- 0.003: Solubility of oxygen in plasma (mL O_2 per mmHg per dL).

Fick's Principle

Used to calculate oxygen consumption (VO_2):

$VO_2 = Q \times (CaO_2 - CvO_2)$

- Q: Cardiac output (L/min).
- CaO_2: Arterial Oxygen Content (mLO_2/dL)
- CvO_2: Venous Oxygen Content (mLO_2/dL)

The equation can be rearranged as:

$Q = VO_2 / (CaO_2 - CvO_2)$

Oxygen Delivery (DO_2)

This calculates the total amount of oxygen delivered to tissues per minute (Normal: 900 - 1100 mL O_2/min)

$DO_2 = Q \times CaO_2$

- DO_2: Oxygen delivery (mL O_2/min).
- Q: Cardiac output (L/min, converted to mL/min by multiplying by 1000).
- CaO_2: Arterial oxygen content (mLO_2/dL)

Oxygen Uptake (VO_2)

Measures oxygen uptake by tissues (Normal: 250-300 mL O_2/min)

$VO_2 = VE \times (FIO_2 - FEO_2)$

- VO_2 = Oxygen uptake (mLO_2/min).
- VE = Minute ventilation (L/min).
- FIO_2 = Fraction of inspired oxygen (e.g. 0.21 for room air).
- FEO_2 = Fraction of expired oxygen (typically 0.16 - 0.17 in normal breathing).

Oxygen Extraction Ratio (OER)

This measures the proportion of oxygen that is extracted by tissues (Normal: 22-30%)

$OER = CaO_2 - CvO_2 / CaO_2$

- CaO_2: Arterial oxygen content.
- CvO_2: Venous oxygen content.

Alternatively, OER can be expressed using oxygen saturations directly:

$OER = SaO_2 - SvO_2 / SaO_2$

Carbon Dioxide Output (VCO_2)

Measures the amount of CO_2 produced and exhaled per minute (Normal: 2-3 $mLCO_2/kg/min$)

$VCO_2 = VE \times (FECO_2 - FICO_2)$

- VCO_2 = Carbon dioxide output ($mL\ CO_2/min$).
- VE = Minute ventilation (L/min).
- $FECO_2$ = Fraction of expired CO_2 (typically 0.04 or 4% in normal breathing).
- $FICO_2$ = Fraction of inspired CO_2 (~0, since inhaled CO_2 is negligible in room air).

Shunt Fraction (Qs/Qt)

Estimates the fraction of blood bypassing ventilated alveoli, often used in hypoxemia (Normal < 5%)

$Qs / Qt = CcO_2 - CaO_2 / CcO_2 - CvO_2$

- Qs/Qt: Shunt fraction (unitless).
- CcO_2: Oxygen content in capillary blood (mLO_2/dL).
- CaO_2: Arterial oxygen content (mLO_2/dL).
- CvO_2: Venous oxygen content (mLO_2/dL).

Respiratory Quotient (RQ)

Describes the ratio of carbon dioxide production to oxygen consumption (Normal: 0.8)

$RQ = VCO_2 / VO_2$

- RQ: Respiratory quotient (unitless).
- VCO_2: Rate of carbon dioxide production (mL/min).
- VO_2: Rate of oxygen consumption (mL/min).

Ventilation-Perfusion Ratio (V/Q)

Describes the relationship between alveolar ventilation and pulmonary blood flow (Normal: 0.8)

$V/Q = VA / Q$

- VA = Alveolar ventilation (L/min).
- Q = Pulmonary blood flow (L/min), equivalent to cardiac output (Q)

Minute Ventilation (VE)

Total volume of air moved in and out of the lungs per minute (Normal: 5 – 8 L/min)

$VE = VT \times RR$

- VT : Tidal volume (L).
- RR: Respiratory rate (breaths per minute).

Alveolar Ventilation (VA)

Volume of air reaching the alveoli per minute, excluding dead space (Normal 4 – 5 L/min)

$VA = (VT - VD) \times RR$

- VT: Tidal volume (L).
- VD: Dead space volume (L).
- RR: Respiratory rate (breaths per minute).

Alveolar Ventilation Equation

Measures efficiency of ventilation (Normal 4 – 5 L/min)

$VA = VCO_2 \times k / PACO_2$

- VA = Alveolar ventilation (L/min).
- $VCO_2 = CO_2$ production (mL/min).
- $PACO_2$ = Alveolar CO_2 partial pressure (mmHg).
- $k = 0.863$ (Constant)

Dead Space (VD) Bohr Equation

Estimates physiological dead space (Normal 20 - 35%)

$VD/VT = PaCO_2 - PECO_2$

- $PaCO_2$: Arterial partial pressure of CO_2.
- $PECO_2$: Mixed expired partial pressure of CO_2.

Dead Space Volume (VD) Fowler's Equation

Measures anatomical dead space (Normal: 2.2 ml/kg IBW)

$VD = VT \text{ x } PaCO_2 - PECO_2 / PaCO_2$

- VT: Tidal volume.
- $PaCO_2$: Arterial partial pressure of CO_2.
- $PECO_2$: Expired CO_2 partial pressure.

Capnography Equation (End-Tidal CO_2)

Measures CO_2 pressure in exhaled breath (Normal 30 – 40 mmHg)

$ETCO_2 = PaCO_2 - (VD / VT \times PaCO_2)$

- $ETCO_2$ = End-tidal CO_2.
- VD/VT = Dead space fraction.
- $PaCO_2$ = Arterial CO_2.

Airway Resistance (RAW)

Quantifies resistance to airflow in the respiratory system (Normal: 0.5 – 2.5 cmH_2O/L/sec)

$Raw = \Delta P / \dot{V}$

- Raw: Airway resistance (cmH_2O/L/sec).
- ΔP: Pressure difference across the airway (cmH_2O).
- $\dot{V}$: Airflow (L/sec).

Respiratory System Compliance (CRS)

Describes the distensibility of the lungs and chest wall (Normal: 70 – 100 mL/cmH_2O)

$C = \Delta V / \Delta PC$

- C: Compliance (mL/cmH_2O).
- ΔV (Delta V): Change in volume (mL).
- ΔP (Delta P): Change in airway pressure (cmH_2O).

Chest Wall Compliance (CCW)

Describes the distensibility of the lungs and chest wall (Normal: 150 - 200 mL/cmH_2O)

$C = \Delta V / \Delta PC$

- C: Compliance (mL/cmH_2O).
- ΔV (Delta V): Change in volume (mL).
- ΔP (Delta P): Change in pleural (esophageal) pressure (cmH_2O).

Lung Compliance (CL)

Describes the distensibility of the lungs and chest wall (Normal: 150 - 200 mL/cmH_2O)

$C = \Delta V / \Delta PC$

- C: Compliance (mL/cmH_2O).
- ΔV (Delta V): Change in volume (mL).
- ΔP (Delta P): Change in trans-pulmonary pressure (cmH_2O).

Time Constant (τ)

Time required for inflation or deflation up to 63% of the final volume (Normal 0.8 – 1.2 seconds)

$\tau = C_{RS} \times R_{AW}$

- τ = Time constant (seconds).
- C_{RS} = Respiratory system compliance (L/cmH_2O).
- R_{AW} = Airway resistance ($cmH_2O/L/s$).

Diffusing Capacity (DLCO)

Represents the lung's ability to transfer gas (Normal 75% - 140% of predicted value)

$DLCO = \dot{V}CO / \Delta PCO$

- $\dot{V}CO$: Uptake of carbon monoxide (mL/min).
- ΔPCO: Partial pressure difference of carbon monoxide across the alveolar-capillary membrane.

Peak Expiratory Flow Rate (PEFR)

A clinical measure of airway obstruction (Normal: 400 – 700 L/min)

$PEFR = \Delta V / \Delta t$

- PEFR: Peak Expiratory Flow rate (L/min)
- ΔV: Volume of air exhaled.
- Δt: Time taken to exhale that volume.

Oxygen-Hemoglobin Dissociation Equation

Estimates oxygen binding to hemoglobin:

$SO_2 / 1 - SO_2 = 10^{(PO2 - P50) / n}$

- SO_2: Oxygen saturation.
- PO_2: Partial pressure of oxygen.
- P50: PO_2 at 50% saturation (typically ~26.6 mmHg).
- n: Hill coefficient (~2.7 for hemoglobin).

Pulmonary Vascular Resistance (PVR)

Calculates resistance in the pulmonary circulation (Normal: 30 - 180 dyn/s/cm-^{5})

$PVR = P_{PA} - P_{LA} / Q$

- PVR: Pulmonary vascular resistance (dyn·s·cm^{-5}).
- P_{PA}: Mean pulmonary artery pressure (mmHg).
- P_{LA}: Left atrial pressure or pulmonary capillary wedge pressure (mmHg).
- Q: Cardiac output (L/min).

Systemic Vascular Resistance (SVR)

Calculates resistance in the systemic circulation (Normal: 800 and 1200 dynes·sec/$^{cm-5}$)

$SVR = P_{SA} - P_{RA} / Q$

- SVR: Systemic vascular resistance (dyn·s·cm^{-5}).
- P_{SA}: Mean systemic artery pressure (mmHg).
- P_{RA}: Right atrial pressure or central venous pressure (mmHg).
- Q: Cardiac output (L/min).

Acid – Base Equations

Henderson-Hasselbalch Equation

Explains the relationship of HCO_3 and $PaCO_2$ to maintain pH

$pH = pKa + \log ([HCO_3] / (0.03 \times PaCO_2)$

- pKa: 6.1 (for bicarbonate buffer system).
- $[HCO_3]$: Bicarbonate concentration (mEq/L).
- $PaCO_2$: Arterial CO_2 pressure (mmHg).
- 0.03 = Solubility constant of CO_2 in plasma.

Respiratory Compensation for Metabolic Disorders

Metabolic Acidosis (Winter's formula):

$PaCO_2 = (1.5 \times HCO_3) + 8 \pm 2$

Predicts expected $PaCO_2$ in metabolic acidosis
If actual $PaCO_2 >$ expected, there is a concurrent respiratory acidosis, if $<$ there is concurrent respiratory alkalosis.

For Metabolic Alkalosis:

$PaCO_2 = (0.7 \times HCO3) + 20 \pm 5$

Predicts expected $PaCO_2$ in metabolic alkalosis.
If $PaCO_2$ is $<$ than expected, there is concurrent respiratory alkalosis, if $>$ there is concomitant respiratory acidosis.

Metabolic Compensation for Respiratory Disorders

Acute Respiratory Acidosis (↑$PaCO_2$):

$\Delta HCO_3 = \Delta PaCO_2 / 10 \times 1$

For every 10 mmHg ↑ in $PaCO_2$, HCO_3 increases by 1 mEq/L (hours).

Chronic Respiratory Acidosis (↑$PaCO_2$):

$\Delta HCO_3 = \Delta PaCO_2 / 10 \times 3-4$

For every 10 mmHg ↑ in $PaCO_2$, HCO_3 increases by 3-4 mEq/L (days).

Acute Respiratory Alkalosis (↓$PaCO_2$):

$\Delta HCO_3 = \Delta PaCO_2 / 10 \times 2$

For every 10 mmHg ↓ in $PaCO_2$, HCO_3 decreases by 2 mEq/L (hours).

Chronic Respiratory Alkalosis (↓$PaCO_2$):

$\Delta HCO_3 = \Delta PaCO_2 / 10 \times 4-5$

For every 10 mmHg ↓ in $PaCO_2$, HCO_3 decreases by 4-5 mEq/L (days).

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Disorder	Primary Change	Compensation	Expected Compensation
Respiratory Acidosis	↑PCO_2	Kidneys retain HCO_3 excrete H^+	Acute: ΔHCO_3 = +1 mEq/L per 10 mmHg ΔPCO_2 Chronic: ΔHCO_3 = +3.5 mEq/L per 10 mmHg ΔPCO_2
Respiratory Alkalosis	↓PCO_2	Kidneys excrete HCO_3 retain H^+	Acute: ΔHCO_3 = -2 mEq/L per 10 mmHg ΔPCO_2 Chronic: ΔHCO_3 = -4-5 mEq/L per 10 mmHg ΔPCO_2
Metabolic Acidosis	↓HCO_3	Lungs hyperventilate to ↓PCO_2	$PCO_2 = (1.5 \times HCO_3) + 8 \pm 2$ (Winter's Formula)
Metabolic Alkalosis	↑HCO_3	Lungs hypoventilate to ↑PCO_2	ΔPCO_2 = +0.7 mmHg per 1 mEq/L ΔHCO_3

Table 1: Summary of Compensation Mechanisms.

Anion Gap (AG) Calculation for Metabolic Acidosis

$AG = Na - (Cl + HCO_3)$

- Na^+ = Serum sodium (normal: 135–145 mEq/L).
- Cl^- = Serum chloride (normal: 96–106 mEq/L).
- $HCO3^-$ = Serum bicarbonate (normal: 22–26 mEq/L).
- Normal Anion Gap = 8–12 mEq/L.

Increased AG (>12 mEq/L) = High Anion Gap Metabolic Acidosis

Normal AG (8–12 mEq/L) **=** Non-Anion Gap Metabolic Acidosis

Corrected Anion Gap for Albumin

$AG\ corrected = AG + 2.5 \times (4 - Albumin)$

Delta-Delta Equation (For Mixed Acid-Base Disorders)

$\Delta AG / \Delta HCO3$

- ΔAG = (Measured AG - Normal AG).
- ΔHCO_3 = (Normal HCO_3 - Measured HCO_3).

If $\Delta AG / \Delta HCO3 \approx 1$ Pure AG metabolic acidosis.

> 1 Concurrent metabolic alkalosis.

< 1 Concurrent normal AG metabolic acidosis.

Strong Ion Difference (SID)

Stewart's Acid-Base Approach to assess metabolic acidosis/alkalosis. (Normal 40 – 50 mEq/L)

$SID = (Na + K + Ca + Mg) - (Cl + \text{other anions})$

- SID: Strong ion difference (mEq/L).
- Na: Serum Sodium (135 – 145 mEq/L).
- K: Serum Potassium (3.5 – 5 mEq/L).
- Ca: Serum Calcium (4.4 – 5.2 mEq/L) (8.8 – 10.4 mg/dL).
- Mg: Serum Magnesium (1.3 – 2.1 mEq/L).
- Cl: Serum Chloride (95 – 105 mEq/L).

Base Excess (BE)

$BE = 0.93 \times (HCO_3 - 24.4) + 14.83 \times (pH - 7.4)$

- BE: Base Excess.
- HCO_3: serum Bicarbonate.
- 0.93: Constant for the buffering capacity of HCO3.
- 14.83: Constant for the buffering capacity of hemoglobin and other plasma proteins.

Determines how much base/acid is needed to return pH to normal.
$BE > 2$ = Metabolic alkalosis, $BE < -2$ = Metabolic acidosis.

Laws of Physics

Boyle’s Law (Pressure-Volume Relationship)

$P_1 V_1 = P_2 V_2$

- P: Pressure.
- V: Volume.

Boyle's Law, in the context of the lungs, describes the inverse relationship between the volume of the lungs and the pressure within them.

Dalton’s Law of Partial Pressures

$P_{gas} = F_{gas} \times P_{total}$

- P_{gas}: Partial pressure of a gas.
- F_{gas}: Fraction of the gas in the mixture.
- P_{total}: Total atmospheric pressure.

Dalton's Law states that the total pressure of the air is the sum of the partial pressures of each of these gases. Each gas contributes to the overall atmospheric pressure in proportion to its concentration in the air.

Henry’s Law (Gas Dissolution in Blood)

$C = kP$

- C: Concentration of dissolved gas.
- K: Henry’s constant.
- P: Partial pressure of gas.

Henry's Law states that the amount of a gas that dissolves in a liquid is directly proportional to the partial pressure of that gas above the liquid, provided the temperature remains constant.

Fick's First Law of Diffusion

$V = AD (P_1 - P_2) / T$

- V: Gas diffusion rate.
- A: Surface area.
- D: Diffusion coefficient.
- $P_1 - P_2$: Pressure difference.
- T: Thickness of the membrane.

Fick's First Law of Diffusion states that the rate of diffusion of a substance is proportional to the concentration gradient and the area available for diffusion.

Poiseuille's Law

$Q = (\pi \Delta P r^4) / (8 \eta l)$

- Q: Flow rate
- ΔP: Pressure difference
- R: Radius of the tube
- H: Viscosity of the fluid
- L: Length of the tube

Poiseuille's Law explains the factors affecting airway resistance in regulating airflow in the lungs.

Laplace's Law

$T = Pr$

- T: Wall tension
- P: Pressure
- r: Radius of the cylinder

Laplace's Law states that the tension in the wall of a sphere is proportional to the product of the pressure inside the sphere or cylinder and the radius of the sphere

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Ideal Gas Law

$PV = nRT$

- P: Pressure
- V: Volume
- N: Number of moles of gas
- R: Ideal gas constant
- T: Temperature (in Kelvin)

The Ideal Gas Law explains how changes in volume, temperature, and pressure are related to each other.

Bernoulli's Principle

$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

- ρ: Density of the fluid
- P: Pressure
- V: Velocity of the fluid
- g: Acceleration due to gravity
- h: Height

Bernoulli's principle explains the relationship between the pressures, density and velocity of fluids or gas.

Hooke's Law

$F = -kx$

- F: Force
- K: Spring constant (lung stiffness)
- x: Displacement from equilibrium (change in volume)

Hooke's Law describes the elastic behavior of the lungs, the balance between the forces that expand the lungs (muscle contraction) and the elastic recoil forces determines the volume of air that moves in and out of the lungs.

Conclusion

The study of respiratory, cardiac, and acid-base equations is not merely an academic exercise but a fundamental aspect of understanding human physiology and pathology. These equations provide a quantitative lens through which we can analyze the intricate interplay between the respiratory, cardiovascular, and metabolic systems, each of which is essential for maintaining homeostasis. From assessing oxygen delivery and carbon dioxide elimination to evaluating cardiac performance and acid-base balance, these mathematical tools empower clinicians and researchers to diagnose, monitor, and treat a wide array of conditions with precision and insight.

As we continue to advance our understanding of human health and disease, the principles and equations discussed in this chapter remain indispensable. They not only deepen our comprehension of normal physiological processes but also provide a foundation for identifying and addressing pathological deviations. By integrating these concepts into clinical reasoning, we are better equipped to improve patient outcomes and advance the field of medicine. Ultimately, mastering these equations is a step toward unraveling the complexities of the human body and harnessing that knowledge to promote health and healing.

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